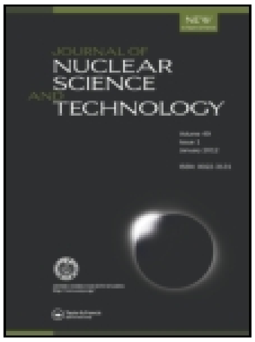

Journal of Nuclear Science and Technology

ISSN: 0022-3131 (Print) 1881-1248 (Online) Journal homepage: https://www.tandfonline.com/loi/tnst20

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Journal of NUCLEAR SCIENCE and TECHNOLOGY, Vol. 40, No. 4, p. 192–200 (April 2003)

Calculation of Gamma-Ray Buildup Factors up to Depths of 100 mfp by the Method of Invariant Embedding, (II)

Improved Treatment of Bremsstrahlung

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(Received December 13, 2002 and accepted in revised form January 23, 2003)

An improved method to calculate the gamma-ray buildup factors including bremsstrahlung has been developed. The exposure buildup factors with bremsstrahlung were computed by the present method for lead, iron and water at the source energy of 10.0 MeV up to depths of 100 mfp. The accuracy of the present method was checked by comparison with the calculations by use of EGS4. Excellent agreement was obtained between the calculations by both methods about the exposure buildup factors per energy (energy spectrum of transmitted photons) for lead up to depths of 10 mfp and the ratio of the exposure buildup factor with bremsstrahlung to that without bremsstrahlung for lead, iron and water up to depths of 40 mfp. It is confirmed that the present method has an accuracy sufficient to be used to the generation of an improved set of gamma-ray buildup factors including bremsstrahlung.

KEYWORDS: bremsstrahlung, buildup factor, radiation transport, radiation shielding, gamma ray, point isotropic source, Monte Calro method, invariant embedding method, multiple scattering

I. Introduction

The present work has been made to generate an improved set of gamma-ray buildup factors for point isotropic sources in infinite homogeneous media based on the method of invariant embedding (IE method). The objectives of the work are as follows:

(1) Extension of the buildup factor up to depths of 100 mean free paths (mfp)
(2) Estimation of error associated with the radiation transport method
(3) Improved treatment of bremsstrahlung
(4) Consistent use of the cross section set.

The results of the work performed according to the objectives (1) and (2) were presented in the previous paper.[1)] The exposure buildup factors for water, iron and lead for typical source energies of 10 MeV, 1.0 MeV and 0.1 MeV were calculated without bremsstrahlung up to depths of 100 mfp by the IE method. Those buildup factors were found to agree well with other existing data up to depths of 40 mfp including the moments method calculations and the Monte Calro calculations by EGS4. A comprehensive survey of buildup factors was performed to estimate errors due to energy, angle and space meshes adopted in the transport calculations by the IE method. It was confirmed that the buildup factors are obtainable up to depths of 100 mfp with an error less than 10% by using the IE method.

The purpose of this article is to present the results of the work performed according to the objective (3): Improved treatment of bremsstrahlung. The work including a generation of an improved set of buildup factors based upon consistent use of the cross section set is in progress, and will be presented in the article to be followed.

A pair of electron and positron created by the pair production reaction and electron recoiled by the Compton scattering lose some of their kinetic energy by emitting photons (bremsstrahlung) as they traverse medium.

Calculations of gamma-ray buildup factors including bremsstrahlung were performed by Takeuchi *et al.*[2)] using the discrete ordinates integral transport code PALLAS and by Subbaiah *et al.*[3)] using the discrete ordinates transport code ASFIT. The treatment of bremsstrahlung in these calculations is based on a simple physical model about the angular distribution of secondary photons emitted through bremsstrahlung. It is assumed that all the secondary photons are emitted in the same direction as the primary photon in the calculation by PALLAS and that all the secondary photons are emitted isotropically in the calculation by ASFIT. The buildup factors for high-Z materials in the standard data compiled by American Nuclear Society[4)] were computed by using the PALLAS code.

A more rigorous treatment of bremsstrahlung in calculating buildup factors was made by Hirayama *et al.*[5)] and Hirayama.[6)] He calculated the gamma-ray buildup factors by using the EGS4 code, that simulates the coupled transport of electron and photon taking into account their multiple scattering with atoms in the medium. He pointed out that the contribution of bremsstrahlung to the exposure buildup factor for lead at 10 MeV in the ANS standard is overestimated and subject to an improvement.

Kitsos *et al.*[7)] calculated exposure buildup factors including bremsstrahlung up to depths of 30 mfp by using the SN1D code based on the discrete ordinate method. Their treatment for bremsstrahlung is based on the analytical expression for the angular distribution of electrons scattered on a thick target derived from the angular distribution of electrons scattered on a thin target.

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Recently, the exposure buildup factors including bremsstrahlung for several materials were calculated up to depths of 60 mfp by Chibani[8] using the Monte Carlo code EBUF with an exponential transform method as a variance reduction technique.

In the present paper, we develop an improved method to calculate the gamma-ray buildup factors including bremsstrahlung. The method is checked by comparing the buildup factors and the energy spectrum of photons obtained by the present method with those obtained by the direct calculations by using the EGS4 code including the coupled transport of both electron and photon. Also presented are calculations of the exposure buildup factors including bremsstrahlung for water, iron and lead at the source energy of 10 MeV up to the depths of 100 mfp.

II. Method of Calculation

1. Outline of Method

The present improved method to calculate the gamma-ray buildup factors including bremsstrahlung is based on combining the transport calculation for electron and positron by using the EGS4 code with the transport calculation of photon by the IE method. The energy and angular distribution of the secondary photons emitted through bremsstrahlung of electron and positron created by the pair production reaction or bremsstrahlung of electron recoiled by the Compton scattering is calculated by using the EGS4 code. Attenuation of photon is cut off in these calculations. This type of calculation using the EGS4 code was first made by Hirayama.[9] Then, the yield of the secondary photons obtained is used to compose the double-differential cross section $\Sigma_b(E_0,\ \omega_0\to E,\ \omega)$ representing the yield of the secondary photons with energy E in the direction ω due to the primary photon with energy E_0 in the direction ω_0, provided that the secondary photons are emitted where the pair production or the Compton scattering takes place. Then, the transport calculations of photons is performed by the IE method with the double-differential cross section obtained.

The significance of the present method as compared with the direct use of the EGS4 code for the coupled transport calculations of both electron and photon is as follows. Firstly, the Monte Calro method is not free from an error due to the variance reduction technique applied to transport calculations of photon at deep penetration. The calculations of buildup factors up to depths of 40 mfp were performed by using the EGS4 code with the particle splitting technique,[6] that should be verified by comparison with the buildup factors obtained by other dependable method, such as the IE method. Secondly, the IE method is able to compute efficiently the buildup factors at very deep penetration, such as at depths of 100 mfp or more. The execution time required to compute the buildup factors up to depths of 100 mfp with an error less than 10% is about 20 min. by a personal computer with the Pentinum-II processor of 500 MHz.

2. Calculation of Bremsstrahlung Yield

Suppose that a pair production reaction occurs with an incident photon with energy E_0 in a homogeneous medium. We define the yield of the secondary photons per pair production reaction $g^p(E,\ x,\ E_0)$ so that $\sigma_{\rm pair}(E_0)g^p(E,\ x,\ E_0)dEdxd\phi$ represents the number of the secondary photons emitted with energy E in the range dE in the direction specified by x (the cosine of the angle between the primary photon and the secondary one) and ϕ (the azimuthal angle between the primary photon and the secondary one) in the range $dxd\phi$ through bremsstrahlung of electron and positron during their traverse in the medium and the annihilation of positron. The yield $g^p(E,\ x,\ E_0)$ is called the bremsstrahlung yield per pair production, although it includes conventionally the secondary photons emitted through the annihilation of positron. The bremsstrahlung yield per Compton scattering $g^c(E,\ x,\ E_0)$ is defined in the same way.

These yields are computed by using the EGS4 code taking into account the multiple scattering of electron and positron with atoms in the medium.[9] In these calculations, the detector for the secondary photons is set to locate at very large distance from the point of reaction so that the range of electrons and positrons transported in the media is negligibly small compared with the distance between the detector and the point of reaction. The attenuation of the secondary photons in the medium is suppressed.

3. Calculation of Double-Differential Cross Section

The bremsstrahlung yield defined in the previous section is used to compose the macroscopic double-differential cross section for the secondary photons through bremsstrahlung and annihilation of positron $\Sigma_b(E_0, \omega_0\to E, \omega)$ from E_0 (energy of the primary photon) to E (energy of secondary photon) and from ω_0 (the direction of the primary photon) to ω (the direction of the secondary photon). The angular variable ω_0 or ω is defined as the cosine of the angle between the direction of photon and the axis perpendicular to a slab. The calculation of the differential cross section is performed based upon an approximation that the secondary photons are emitted at the point where the pair production or the Compton scattering takes places. This approximation is valid when the range of electron and positron is negligible compared with the mean free path of secondary photon and will be discussed later.

The microscopic double-differential cross section for bremsstrahlung and annihilation of positron due to the pair creation is expressed as

$$\sigma_{\rm pair}(E_0, \omega_0 \to E, \omega) = \int_0^{2\pi} d\phi\sigma_{\rm pair}(E_0)g^p(E, x, E_0). \quad (1)$$

By changing the variable from ϕ (the azimuthal angle between the primary photon and the secondary one) to x (the cosine of the angle between the primary photon and the secondary one) by using the equation expressing the variable x in terms of the variables ω, ω_0 and ϕ,

$$x = \sqrt{(1-\omega^2)(1-\omega_0^2)}\cos\phi + \omega\omega_0, \quad (2)$$

Eq. (1) can be reduced to

$$\sigma_{\rm pair}(E_0,\omega_0 \to E,\omega) = \int_{x^-}^{x^+} dx \frac{2\sigma_{\rm pair}(E_0) g^p(E,\ x,\ E_0)}{\sqrt{(x-x^-)(x^+-x)}}, \tag{3}$$

where

$$x^{\pm} = \omega\omega_0 \pm \sqrt{1-\omega^2)(1-\omega_0^2)}. \tag{4}$$

The integral with respect to the variable x in Eq. (3) is performed analytically in the actual calculation based on the approximation that the yield $g^p(E,\ \omega,\ E_0)$ is unchanged with x within an angle bin used in the Monte Carlo calculation by EGS4.

It can be proved mathematically that the following equation holds

$$\int_{-1}^{1} d\omega\sigma_{\rm pair}(E_0,\omega_0 \to E,\omega) = 2\pi\sigma_{\rm pair}(E_0)\int_{-1}^{1} dx g^p(E,\ x,\ E_0). \tag{5}$$

It should noted that the right hand side of Eq. (5) is independent of the variable ω_0 reflecting the physical fact that the cross section integrated over the solid angle of the secondary photon is independent of the direction of the primary photon. Equation (5) is used as a sum rule to check the computed double-differential cross section.

Similarly, the microscopic double-differential cross section for bremsstrahlung due to the Compton scattering is given as

$$\sigma_{comp}(E_0,\ \omega_0 \to E,\omega) = \int_{x^-}^{x^+} dx \frac{2\sigma_{comp}(E_0) g^c(E,\ x,\ E_0)}{\sqrt{(x-x^-)(x^+-x)}}. \tag{6}$$

It also satisfies a sum rule similar as Eq. (5).

Then, the desired macroscopic double-differential cross section for bremsstrahlung and annihilation of positron $\Sigma_b(E_0,\omega_0 \to E,\omega)$ can be obtained.

A transport calculation of photon by the IE method is actually performed based on the multi-group approximation with respect to energy and the discrete ordinate approximation with respect to the angular variable. The double-differential cross section to be used in the transport calculation of photon is discretized as

$$\Sigma_{bnm}(\omega_i,j) = \frac{1}{\Delta\omega_j \Delta E_m}\int_{\omega_j^-}^{\omega_j^+} d\omega_0 \int_n dE \times \int_m dE_0 \Sigma_b(E_0,\omega_0 \to E,\omega_i), \tag{7}$$

where ω_j^+ and ω_j^- represent respectively the upper boundary and the lower one of the angular region j adopted in the discrete ordinate approximation and $\Delta\omega_j$, the width of the angular region j. $\int_n dE$ represents the integral over the energy group n and ΔE_m, the width of the group m.

4. Transport Calculation of Photon by IE Method

Transport calculations of photon are performed by the IE method by using the double-differential cross section for bremsstrahlung and annihilation of positron. The outline of the IE method is described in the previous paper.[1] More detailed description is found elsewhere.[10,11]

According to the IE method, the modified transmission function for a slab of the thickness X $\tilde{T}(E,\omega|E_0,\omega_0;X)$ is computed with 80 to 100 energy groups and 13 to 15 angular divisions in the range $0 \le \omega \le 1$. The method of direct numerical integration with respect to the slab thickness X is applied to get solution for the function. The equation for the function is integrated with respect to the initial slab thickness by using the Runge-Kutta method for the first step. The solutions for the extended thickness are obtained by using the functional relation for the function

$$\tilde{T}(E,\omega|E_0,\omega_0;X+X') = \int_0^{E_0} dE' \int_0^1 d\omega' \tilde{T}(E,\omega|E',\omega';X') \times \tilde{T}(E',\omega'|E_0,\omega_0;X). \tag{8}$$

The actual calculation of buildup factors for lead including bremsstrahlung requires the initial slab thickness as small as about 1/130,000 mfp to keep the error due to the spatial mesh negligibly small (less than 0.02%). This thickness is very small compared with the slab thickness of 1/1,024 mfp used for the calculations without bremsstrahlung. The small mesh width required for the case with bremsstrahlung comes from the physical fact that a photon with the energy of 10 MeV can be slowed down to very low energy such as 0.01 MeV through bremsstrahlung, whereas a photon with the same energy can be slowed down only to about 0.25 MeV through a Compton scattering. Since the total cross section of lead at 0.01 MeV is lager than that at 0.25 MeV by the factor about 150, the mesh width to treat photon with the energy of 0.01 MeV should be reduced by the factor about 150. The calculation can easily be extended to a large depth by the IE method even starting from the very small mesh width of 1/130,000 mfp, since the solution for the thickness doubled can be obtained successively by using the functional relation (8).

The gamma-ray buildup factors for a point isotropic source in an infinite homogeneous medium can then be obtained from the solution for the modified transmission function and the reflection function of the semi-infinite medium.

III. Results and Discussion

1. Bremsstrahlung Yield

The bremsstrahlung yield per pair production reaction and that per Compton scattering defined previously are computed for lead, iron and water by using EGS4.[9] Actual Monte Carlo calculations are performed with 43 energy bins for the energy of the secondary photon in the range below 16.0 MeV and with 40 angle bins for the angle between the secondary photon and the primary one in the range 0 to 180°. The bremsstrahlung yield for lead at the primary photon energy of 10.0 MeV is shown in **Figs. 1**, **2** and **3**. Figure 1 gives the yield per pair creation per solid angle per energy bin for some of the energy bins as the function of the angle between the secondary photon and the primary one. Figure 2 gives the yield per solid angle per pair creation and that per Compton scattering integrated over the energy of the secondary photons. Figures 1 and 2 indicate that the angular distribution

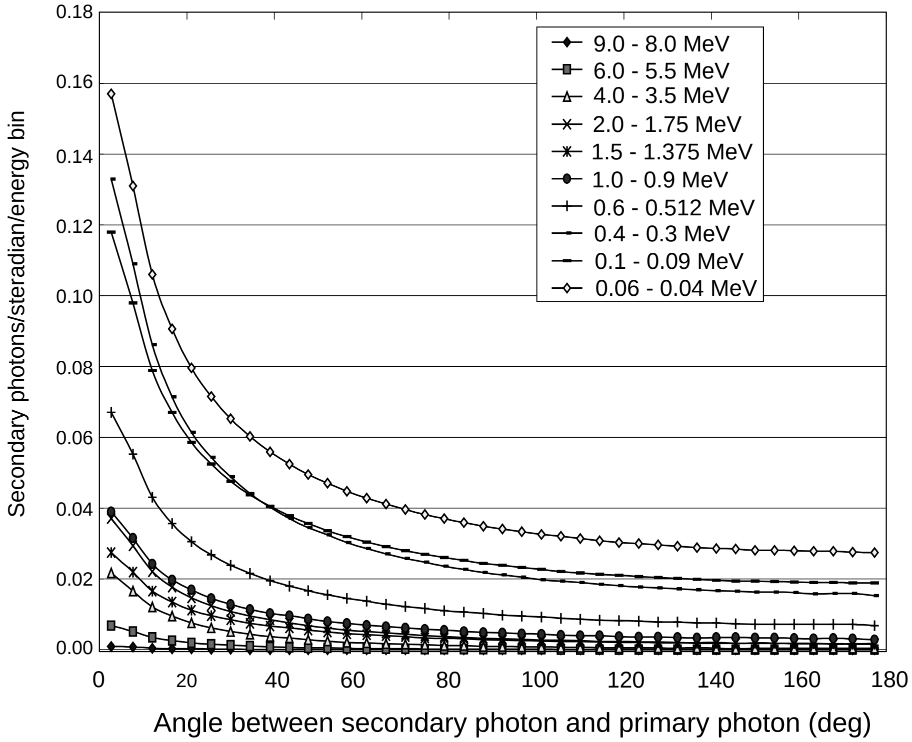

Fig. 1 Yield of secondary photons per energy bin per pair creation emitted through bremsstrahlung and positron annihilation in lead

Energy of primary photon: 10.0 MeV

[9.0–8.0 MeV]: Yield of secondary photons per solid angle per pair creation in an energy bin $8.0 \leq E \leq 9.0$ MeV used in the Monte Carlo calculation

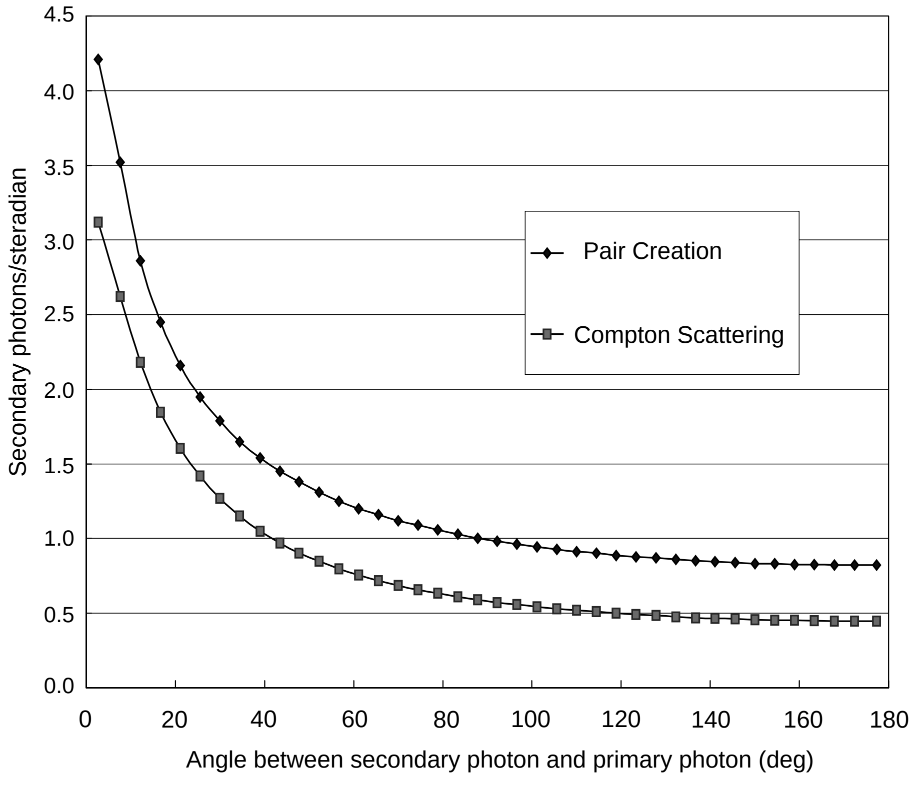

Fig. 2 Yield of secondary photons integrated over energy emitted through bremsstrahlung and positron annihilation in lead

Energy of primary photon: 10.0 MeV

Pair creation: Yield of secondary photons per pair creation

Compton scattering: Yield of secondary photons per Compton scattering

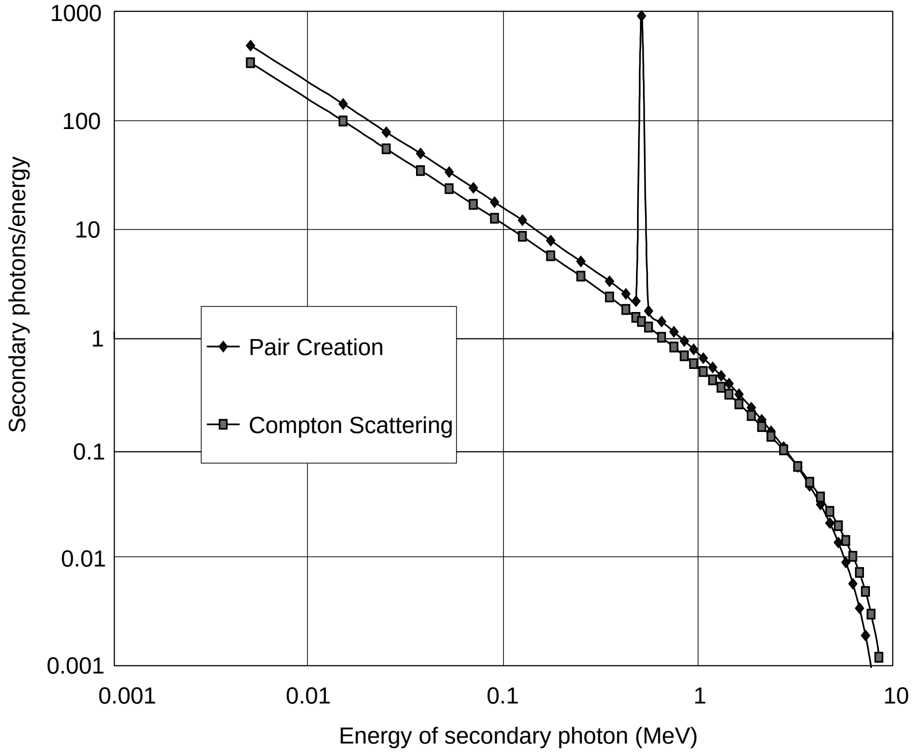

Fig. 3 Yield of secondary photons per energy integrated over solid angle emitted through bremsstrahlung and positron annihilation in lead
Energy of primary photon: 10.0 MeV
Pair creation: Yield of secondary photons per pair creation
Compton scattering: Yield of secondary photons per Compton scattering

of the secondary photons obtained has the isotropic component as well as the peak in the direction of the primary photon (angle 0 in Figs. 1 and 2). The real angular distribution is consistent neither with the simple model used in the PALLAS (all the secondary photons are emitted in the direction of the primary photon), nor with the model used in the ASFIT (all the secondary photons are emitted isotropically), but takes its position at an intermediate point between both simple models. Figure 3 gives the yield of the secondary photons per energy integrated over the solid angle as the function of energy of the secondary photon. It indicates that the yield per energy increases with energy of the secondary photon decreasing.

2. Energy Spectrum of Transmitted Photons

The energy spectrums of gamma-ray including the effect of bremsstrahlung were computed both by the present method and the direct use of EGS4 for the coupled transport calculation of photon and electron. The cross section set PHOTX[12] was used in both calculations. Computations of the energy spectrum were made for gamma-ray transmitted in an infinite homogeneous medium of lead from a point isotropic source with energy of 10.0 MeV at the depths of 1 mfp, 5 mfp and 10 mfp by both methods. **Figure 4** gives the energy spectrum at the depths of 10 mfp. The value in the perpendicular axis of Fig. 4 denoted by exposure buildup factor per energy is defined as the energy spectrum of gamma-ray flux per energy multiplied by the factor $4\pi r^4 \exp(\mu(E)r)(E_0\mu_{enair}(E_0))/(E\mu_{enair}(E))$, where r, E_0, $\mu(E)$ and $\mu_{enair}(E)$ represent respectively the distance from the source, the source energy, the total cross section of the medium and the energy absorption cross section for air, so that its integration with energy gives the exposure buildup factor. The computations of the energy spectrum were made both with and without bremsstrahlung and positron annihilation for comparison. Figure 4 indicates an excellent agreement between the energy spectra computed by both methods. Same results were obtained about the energy spectra at the depths of 1 mfp and 5 mfp. The excellent agreement indicates that both methods are valid in the calculation of gamma-ray buildup factors.

3. Exposure Buildup Factor

The exposure buildup factors including the effect of bremsstrahlung were computed for water, iron and lead at the source energy of 10.0 MeV up to depths of 100 mfp by the present method. The cross section set NBS[13] was used in the calculations for water and iron and the PHOTX set was used in the calculation for lead. The ratio of exposure buildup factor with bremsstrahlung to that without bremsstrahlung for lead is given in **Table 1** and **Fig. 5** together with the ratios of buildup factor obtained by other methods including EGS4[6] with PHOTX set, PALLAS[2] with PHOTX set and ASFIT[5] with NBS set. The same ratio obtained by Kitsos *et al.*[7] behaves almost same as the ratio obtained by ASFIT, although it is not included in Table 1 and Fig. 5, since their result was not given in the digital value. Comparison with the recent data

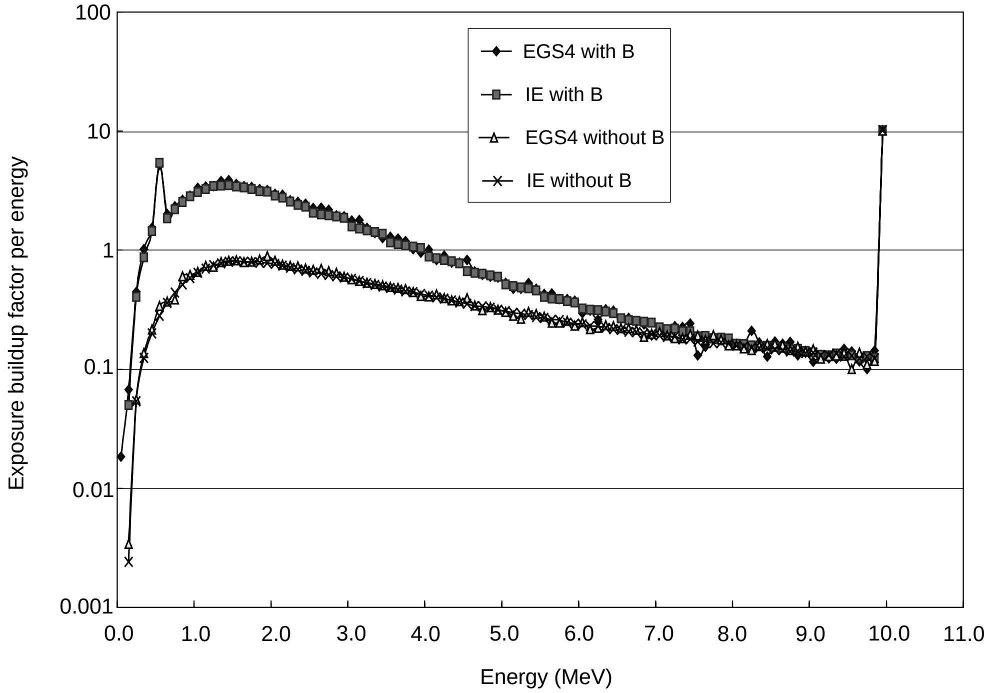

Fig. 4 Exposure buildup factor per energy in lead at depths of 10 mfp
Source energy: 10.0 MeV
EGS4 with B: Calculation by EGS4 with bremsstrahlung and positron annihilation
IE with B: Calculation by the present method with bremsstrahlung and positron annihilation
EGS4 without B: Calculation by EGS4 without bremsstrahlung and positron annihilation
IE without B: Calculation by the present method without bremsstrahlung and positron annihilation

obtained by Chibani[8] is neither included, since the buildup factors without bremsstrahlung was not given by him. Table 1 and Fig. 5 indicate an excellent agreement between the data obtained by EGS4 and the IE method. Both data coincide within a discrepancy less than 2% up to depths of 10 mfp and less than 5% up to depths of 40 mfp. The data obtained by EGS4 and the IE method has the intermediate value between the data by PALLAS and that by ASFIT as expected from the comparison about the angular distribution of the secondary photons. **Table 2** gives the ratio of exposure buildup factors with and without bremsstrahlung for iron and water computed by the present method and EGS4 both based on NBS set. The table indicates an excellent agreement between data by both methods. They coincide within a discrepancy less than 5% for iron and less than 3% for water up to depths of 40 mfp. The excellent agreement indicates both methods are consistent in the treatment of bremsstrahlung.

Table 3 gives the exposure buildup factors for water, iron and lead up to depths of 100 mfp obtained by the present method. The exposure buildup factors in the ANS standard[4] and those obtained by EGS4 up to depths of 40 mfp based on the same cross section set are also given for comparison. Other data obtained by Subbaiah *et al.*[3] and Chibani[8] with the different cross section are not included for comparison, since the absolute value of buildup factor is sensitive to the cross section used. The data by the IE method coincide with the data by EGS4 within a discrepancy less than about 8% for lead, about 12% for iron and about 3% for water up to depths of 40 mfp. This magnitude of discrepancy is almost same as that in the buildup factors without bremsstrahlung confirmed in the previous paper.[1] It was also confirmed in the previous paper that the buildup factors without bremsstrahlung are obtainable up to depths of 100 mfp with an error less than 10% by using the IE method. Since Fig. 5 indicates that the ratio of buildup factor with bremsstranlung to that without bremsstralung varies with depth rather slowly beyond depths of 40 mfp, an error of buildup factor due to the treatment of bremsstrahlung at depths of 100 mfp is estimated same as that at the depths of 40 mfp, that is about 5% or less. Therefore, an error in the calculations of buildup factors with bremsstrhalung by the present method is estimated less than about 15% up to depths of 100 mfp. The error of this magnitude in the calculation of buildup factors is acceptable in the shielding calculation, since the attenuation factor of gamma-ray at the depths of 100 mfp is about $\exp(-100) \approx 10^{-44}$.

4. Effect of Electron Transport

The present method is based on the approximation that the secondary photons are emitted at the point where the pair production or the Compton scattering takes places. To check this approximation, the average radius of electron and positron created by the pair production and transporting in the media before emitting the secondary photon with energy E in the direction x (the cosine of the angle between the primary photon and the secondary one) $R(E, x)$ was computed for lead by using EGS4. Then, the bremsstrahlung yield per pair creation $g^p(E, x, E_0)$ is corrected by the factor $\exp(\mu(E)R(E, x))$, where $\mu(E)$ is the total cross section of the medium for pho-

Table 1 Ratio of exposure buildup factor with bremsstrahlung to that without bremsstrahlung for lead

Source energy: 10.0 MeV

Depth (mfp)	IE[a)]	EGS4[b)]	PALLAS[c)]	ASFIT[d)]	IE/EGS4
1.0	1.308	1.308	1.269		0.999
2.0	1.523	1.534	1.534		0.993
3.0	1.707	1.727	1.789		0.989
4.0	1.869	1.889	2.036		0.989
5.0	2.012	2.040	2.270	1.995	0.986
6.0	2.136	2.172	2.481		0.983
7.0	2.243	2.277	2.661		0.985
8.0	2.332	2.369	2.826		0.984
10.0	2.461	2.484	3.086	2.258	0.991
15.0	2.574	2.638	3.342	2.283	0.976
20.0	2.541	2.608	3.320	2.240	0.974
24.0			3.362	2.182	
25.0	2.469	2.535			0.974
30.0	2.396	2.455	3.095		0.976
35.0	2.331	2.373	2.993		0.982
40.0	2.275	2.179	2.912		1.044
45.0	2.228				
50.0	2.188				
55.0	2.153				
60.0	2.122				
65.0	2.096				
70.0	2.072				
75.0	2.051				
80.0	2.032				
85.0	2.015				
90.0	2.000				
95.0	1.986				
100.0	1.973				

a) Calculation by the present method with PHOTX cross section
b) Caculation by EGS4[6)] with PHOTX cross section
c) Calculation by PALLAS[2)] with PHOTX cross section
d) Calculation by ASFIT[5)] with NBS cross section

Table 2 Ratio of exposure buildup factor with bremsstrahlung to that without bremsstrahlung for iron and water

Source energy: 10.0 MeV

Depth	Iron			Water		
(mfp)	IE[a)]	EGS4[b)]	IE/EGS4	IE[a)]	EGS4[b)]	IE/EGS4
1.0	1.127	1.127	1.000	1.036	1.051	0.986
2.0	1.193	1.199	0.995	1.047	1.065	0.983
3.0	1.233	1.238	0.996	1.055	1.071	0.985
4.0	1.255	1.258	0.997	1.057	1.080	0.979
5.0	1.272	1.277	0.996	1.059	1.075	0.985
6.0	1.278	1.279	1.000	1.056	1.075	0.983
7.0	1.280	1.277	1.003	1.058	1.078	0.981
8.0	1.279	1.270	1.007	1.056	1.072	0.985
10.0	1.275	1.282	0.995	1.056	1.078	0.980
15.0	1.256	1.241	1.012	1.056	1.078	0.980
20.0	1.238	1.202	1.030	1.055	1.066	0.989
25.0	1.218	1.221	0.997	1.053	1.058	0.995
30.0	1.204	1.211	0.995	1.054	1.076	0.979
35.0	1.192	1.209	0.986	1.053	1.034	1.018
40.0	1.186	1.242	0.954	1.055	1.027	1.027
45.0	1.176			1.050		
50.0	1.168			1.053		
55.0	1.167			1.049		
60.0	1.159			1.052		
65.0	1.152			1.055		
70.0	1.151			1.051		
75.0	1.146			1.054		
80.0	1.144			1.051		
85.0	1.140			1.048		
90.0	1.138			1.055		
95.0	1.132			1.053		
100.0	1.130			1.050		

a) Calculation by the present method with NBS cross section
b) Caculation by EGS4[6)] with NBS cross section

ton, and used in the calculation of the energy spectrum of photon flux by the IE method. **Figure 6** gives the exposure buildup factor per energy at the depths of 10 mfp in the medium of lead from a point isotropic source with energy of 10.0 MeV. It is computed with the corrected bremsstrahlung yield for pair creation and compared with that computed with the uncorrected yield. Bremsstralung due to electron recoiled by the Compton scattering is not included in the calculation. Figure 6 indicates both exposure buildup factors per energy coincide well in the energy region above about 0.2 MeV. This is consistent with the computed radius of transport for electron and positron $R(E, x)$ that ranges about 0.1 cm or less and small compared with the mean free path of photon in the energy region above about 0.2 MeV. In the energy region below about 0.2 MeV, the radius of transport for electron and positron becomes comparable or even larger than the mean free path of photon and has some effect upon the energy spectrum of photons. Fortunately, the exposure buildup factor per energy in the energy region below about 0.2 MeV is very small and has a negligible contribution to the buildup factor integrated over the energy. It is, therefore, concluded that the effect of the transport of electron and positron is negligible in the calculation of the exposure buildup factor for lead.

IV. Conclusion

An improved method to calculate the gamma-ray buildup factors including bremsstrahlung has been developed. It consists of the calculation of the bremsstrahlung yield by using the EGS4 code taking into account the multiple scattering of electron and positron in the medium and the transport calculation of photon by the invariant embedding method. The exposure buildup factors with bremsstrahlung were computed by the present method for lead, iron and water at the source energy of 10.0 MeV up to depths of 100 mfp. The exposure buildup factors per energy (energy spectrum of transmitted photons) for lead up to depths of 10 mfp computed both by the present method and the direct use of EGS4 for the coupled transport of electron and photon agree excellently. The ratio of the exposure buildup factor with bremsstrahlung to that without bremsstrahlung for lead, iron and water computed both by the present method and the direct use of EGS4 also agree excellently. The ratios obtained both methods coincide within a discrepancy of about 5% or

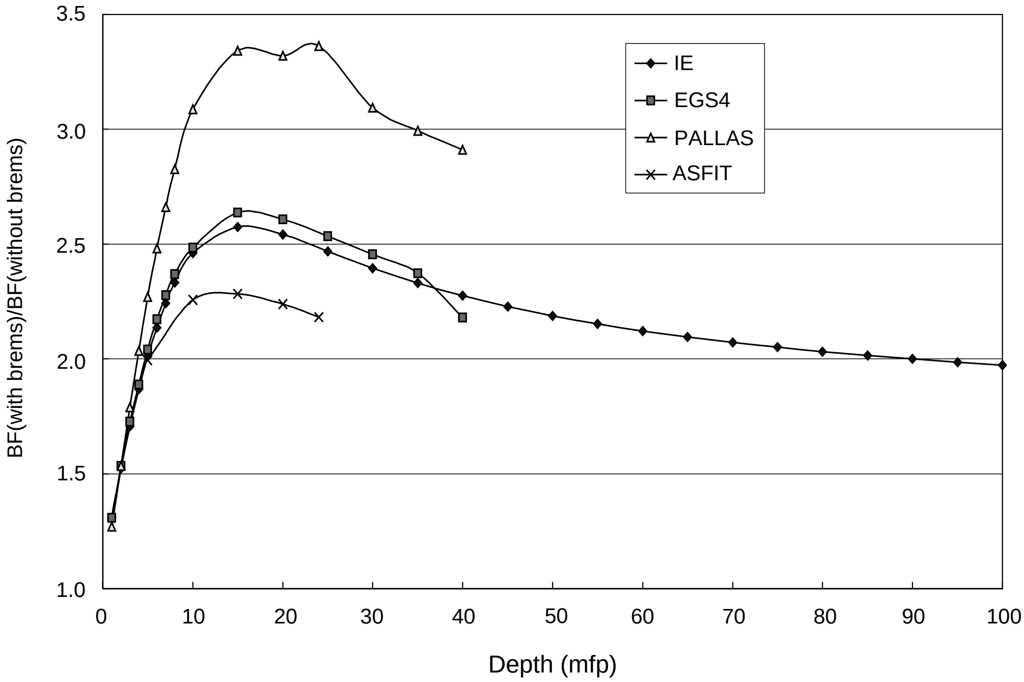

Fig. 5 Ratio of exposure buildup factor with bremsstrahlung to that without bremsstrahlung for lead
Source energy: 10.0 MeV
IE: Calculation by the present method with PHOTX cross section
EGS4: Calculation by EGS4[6] with PHOTX cross section
PALLAS: Calculation by PALLAS[2] with PHOTX cross section
ASFIT: Calculation by ASFIT[5] with NBS cross section

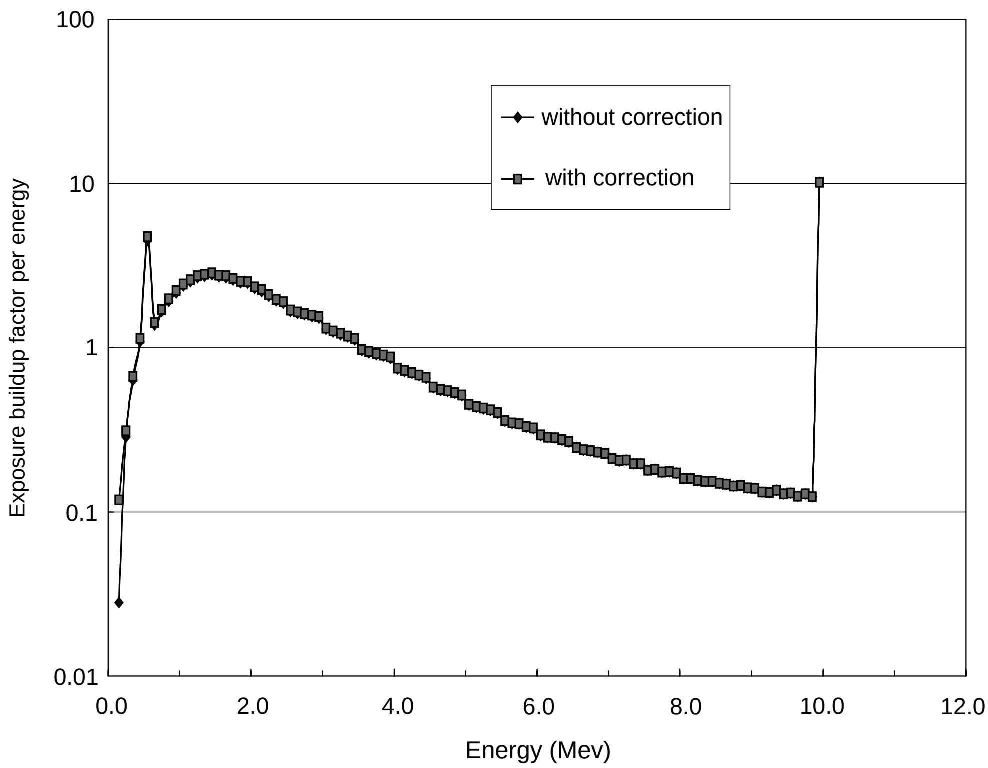

Fig. 6 Exposure buildup factor per energy at depths of 10 mfp in lead calculated with and without correction for transport effect of electron and positron
Source energy: 10.0 MeV
With correction: Calculation with the bremsstrahlung yield per pair creation corrected for transport effect of electron and positron
Without correction: Calculation with the bremsstrahlung yield per pair creation without correction

Table 3 Gamma-ray exposure buildup factors with bremsstrahlung for point isotropic source

Source energy: 10.0 MeV

Depth	Lead				Iron			Water		
(mfp)	IE[a]	EGS4[b]	ANS[c]	IE/EGS4	IE[a]	EGS4[b]	IE/EGS4	IE[a]	EGS4[b]	IE/EGS4
1.0	1.56E+00	1.57E+00	1.51E+00	0.993	1.51E+00	1.51E+00	1.000	1.44E+00	1.44E+00	1.000
2.0	2.02E+00	2.04E+00	2.01E+00	0.990	1.92E+00	1.93E+00	0.995	1.78E+00	1.79E+00	0.994
3.0	2.55E+00	2.59E+00	2.63E+00	0.985	2.33E+00	2.34E+00	0.996	2.10E+00	2.11E+00	0.995
4.0	3.19E+00	3.23E+00	3.42E+00	0.988	2.76E+00	2.78E+00	0.993	2.41E+00	2.43E+00	0.992
5.0	3.97E+00	4.04E+00	4.45E+00	0.983	3.23E+00	3.27E+00	0.988	2.71E+00	2.72E+00	0.996
6.0	4.94E+00	5.04E+00	5.73E+00	0.979	3.72E+00	3.76E+00	0.989	3.00E+00	3.01E+00	0.997
7.0	6.13E+00	6.24E+00	7.37E+00	0.982	4.25E+00	4.29E+00	0.991	3.29E+00	3.31E+00	0.994
8.0	7.61E+00	7.77E+00	9.44E+00	0.979	4.82E+00	4.85E+00	0.994	3.57E+00	3.58E+00	0.997
10.0	1.17E+01	1.20E+01	1.54E+01	0.978	6.07E+00	6.14E+00	0.989	4.13E+00	4.16E+00	0.993
15.0	3.49E+01	3.64E+01	5.08E+01	0.959	9.97E+00	1.01E+01	0.987	5.47E+00	5.54E+00	0.987
20.0	1.04E+02	1.09E+02	1.61E+02	0.953	1.51E+01	1.55E+01	0.974	6.76E+00	6.80E+00	0.994
25.0	3.06E+02	3.27E+02	4.95E+02	0.936	2.18E+01	2.32E+01	0.940	8.00E+00	8.03E+00	0.996
30.0	8.91E+02	9.65E+02	1.47E+03	0.923	3.01E+01	3.22E+01	0.935	9.22E+00	9.45E+00	0.976
35.0	2.56E+03	2.80E+03	4.28E+03	0.913	4.04E+01	4.45E+01	0.908	1.04E+01	1.03E+01	1.010
40.0	7.24E+03	7.65E+03	1.22E+04	0.946	5.30E+01	6.00E+01	0.883	1.16E+01	1.15E+01	1.009
45.0	2.03E+04				6.80E+01			1.27E+01		
50.0	5.60E+04				8.57E+01			1.39E+01		
55.0	1.53E+05				1.07E+02			1.50E+01		
60.0	4.16E+05				1.31E+02			1.61E+01		
65.0	1.12E+06				1.59E+02			1.73E+01		
70.0	3.00E+06				1.91E+02			1.84E+01		
75.0	7.95E+06				2.28E+02			1.95E+01		
80.0	2.10E+07				2.70E+02			2.06E+01		
85.0	5.51E+07				3.17E+02			2.17E+01		
90.0	1.44E+08				3.70E+02			2.29E+01		
95.0	3.74E+08				4.29E+02			2.40E+01		
100.0	9.67E+08				4.95E+02			2.51E+01		

[a] Calculation by the present method with PHOTX cross section for lead and NBS cross section for water and iron

[b] Caculation by EGS4[6] with PHOTX cross section for lead and NBS cross section for water and iron

[c] Data in ANS standard[4] calculated by PALLAS with PHOTX cross section

less up to depths of 40 mfp. The excellent agreement indicates that both the present method and the direct use of EGS4 are valid in the calculation of gamma-ray buildup factors including bremsstrahlung. It is confirmed that the present method has an accuracy sufficient to be used to the generation of an improved set of gamma-ray buildup factors including bremsstrahlung.

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